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# The role of a dynamical measure and dynamical tension in brane creation and growth

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## Abstract

The use of a non-Riemannian measure of integration in the action of strings and branes allows the possibility of dynamical tension. In particular, lower dimensional objects living in the string/brane can induce discontinuities in the tension: the effect of pair creation on the string tension is studied. We investigate then the role that these new features can play in string and brane creation and growth. A mechanism is studied by means of which a scalar field can transfer its energy to the tension of strings and branes. An infinite dimensional symmetry group of this theory is discussed. Creation and growth of bubbles in a formulation that requires mass generation for the bulk gauge fields coupled to the branes is also discussed.

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## I. INTRODUCTION

Extended objects are presently playing an important role in theoretical physics, as candidate theories for the unification of all interactions, like in superstring theory or $M$-theory [1]. Also in cosmology, cosmic strings and domain walls have been extensively discussed [2].

One important question related to the dynamics of *extended objects*, is whether the tension of strings and branes could be dynamical.

In [3] a framework has been discussed, in which the tension of an extended objects is not just an *external input*, but appears as an integration constant. This formulation of the theory of strings and branes requires the use of a non-Riemannian measure of integration in the action for the extended object.

It is perhaps worthwhile to remark, that a non-Riemannian integration measure has been already introduced in the context of field theories of particles and gravitation, in order to address the problems of the cosmological constant, of the spontaneous breaking of scale invariance, of the fermion families, of the dark energy as well as in the brane world scenario [4].

In the consistent formulation of string and brane theories with dynamical tension, internal gauge fields have also to be introduced. Then, the integration of the equations of motion derived from the variation of these gauge fields, gives rise to a dynamical string and brane tension. The coupling of the gauge fields to the brane world-currents allows the tension to become discontinuous in the case of point-like sources.

As we will discuss here, a scalar field defined in the bulk can be used to induce an appropriate world-sheet current, that then affects directly the tension of the extended objects it couples to.

This provides a way to directly *pump* the energy, that the scalar field has, into a network of string or branes: such a process can be of interest in cosmology.

From the more formal point of view, it is also of interest to understand what kind of symmetries are possessed by this kind of description of an extended object: we will see that an infinite dimensional set of symmetries is present.

Finally we also discuss the possibility of bubble creation and growth in a formulation where the coupling to external antisymmetric gauge fields is modulated by the same factor that affects the string tension, especially in connection with the necessity of mass generation

for the antisymmetric gauge fields.

## II. STRINGS AND BRANES WITH DYNAMICAL TENSION

There is an intimate connection between the concept of strings and branes with dynamical tension and the possibility to use a modified measure of integration in their action.

Indeed, when performing the integration in the action functional for the extended object, one should use an invariant volume element. The standard (Riemannian) volume element is

$$\sqrt{-\gamma}d^p\sigma, \tag{1}$$

where $\gamma = \det(\gamma_{ab})$ is the determinant of the metric $\gamma_{ab}$ defined on the manifold. Given a reparametrization of the coordinates $\sigma^a$, the volume element (1) is, of course, invariant.

This invariance can be achieved also if, instead of $\sqrt{-\gamma}$, another *density* is used. For example, given $p$ scalars $\varphi_a$, $a = 1, \ldots, p$, one can construct the density

$$\Phi = \epsilon_{a_1 \ldots a_p}\epsilon^{\mu_1 \cdots \mu_p}\partial_{\mu_1}\varphi^{a_1} \ldots \partial_{\mu_p}\varphi^{a_p}, \tag{2}$$

where $\epsilon^{\mu_1 \cdots \mu_p}$ and $\epsilon_{a_1 \ldots a_p}$ are the alternating symbols. With this definition, $\Phi$ transforms exactly as $\sqrt{-\gamma}$ under a reparametrization transformation (which means that $\Phi/\sqrt{-\gamma}$ is a scalar).

A straightforward use of the measure (2) in string theory is somewhat problematic however. Indeed, if in the Polyakov action [1]

$$S_{\mathrm{P}}[X^\alpha, \gamma_{mn}] = -T\int d\sigma^0 d\sigma^1 \sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} \tag{3}$$

we simply replace $\sqrt{-\gamma}$ by

$$\Phi = \epsilon^{ab}\epsilon_{ij}\partial_a\varphi^i\partial_b\varphi^j \tag{4}$$

we obtain the action

$$S_1 = -\int d\sigma^0 d\sigma^1 \Phi\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} \tag{5}$$

(an overall factor $T$ in (5) is *now irrelevant* since it can be eliminated by simply rescaling the $\varphi^j$ fields). However, the action (5) is not satisfactory, since the variation with respect to $\gamma^{ab}$ gives

$$\Phi\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} = 0, \tag{6}$$

which means that either $\Phi = 0$ or that the induced metric on the string vanishes.

To improve the situation and obtain a more satisfactory action, which still uses the measure $\Phi$, we notice that the use of a measure $\Phi$ opens new possibilities for allowed contributions to the action: indeed, let us consider, for instance, the case in which $\sqrt{-\gamma}L$ is a total derivative; then changing the measure, it could certainly be the case that $\Phi L$ is not a total derivative anymore: this is exactly the situation if

$$L = \frac{\epsilon^{ab}}{\sqrt{-\gamma}} F_{ab}, \tag{7}$$

where $F_{ab} = \partial_a A_b - \partial_b A_a$. So, if we consider the action

$$S = S_1 + S_{\text{gauge}}, \tag{8}$$

where $S_1$ is given by (5) and

$$S_{\text{gauge}} = \int d\sigma^0 d\sigma^1 \Phi \frac{\epsilon^{ab}}{\sqrt{-\gamma}} F_{ab}, \tag{9}$$

we can see that (8) is now much more interesting. First, let us note that it is conformally invariant, provided the fields $\varphi^i$ are transformed as

$$\varphi^i \quad \longrightarrow \quad \varphi'^i = \varphi'^i(\varphi^j), \quad \Phi \quad \longrightarrow \quad J\Phi, \tag{10}$$

where $J$ is the Jacobian of the transformation of the $\varphi^i$ fields, and

$$\gamma_{ab} \quad \longrightarrow \quad \gamma'_{ab} = J\gamma_{ab}. \tag{11}$$

Moreover the variation of the action (8) with respect to $\varphi^j$ gives

$$\epsilon^{ab}\partial_b\varphi^j\partial_a\left(-\gamma^{cd}\partial_c X^\mu \partial_d X^\nu g_{\mu\nu} + \frac{\epsilon^{cd}}{\sqrt{-\gamma}} F_{cd}\right) = 0. \tag{12}$$

If $\det(\epsilon^{ab}\partial_b\varphi^j) \neq 0$, which is always true if $\Phi \neq 0$, then (12) means that all the derivatives of the quantity inside the round brackets are zero, i.e. this quantity is a constant:

$$-\gamma^{cd}\partial_c X^\mu \partial_d X^\nu g_{\mu\nu} + \frac{\epsilon^{cd}}{\sqrt{-\gamma}} F_{cd} = M = \text{const.}. \tag{13}$$

Considering then the variation with respect to $\gamma^{ab}$ of the action (8), which *now* is *non trivial*, we obtain

$$-\Phi\left(\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{1}{2}\gamma_{ab}\frac{\epsilon^{cd}}{\sqrt{-\gamma}} F_{cd}\right) = 0. \tag{14}$$

Solving for $F_{cd}/\sqrt{-\gamma}$ from (13) and inserting in (14) we get

$$\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu g_{\mu\nu} - \frac{1}{2}\gamma_{ab} M = 0. \tag{15}$$

The trace of the above equation, however, gives $M = 0$, so that (15) is nothing but the usual equation obtained in string theory with the standard integration measure.

If we now look at the equation of motion obtained from the variation of the gauge field $A_a$, we obtain

$$\epsilon^{ab}\partial_b \left(\frac{\Phi}{\sqrt{-\gamma}}\right) = 0, \tag{16}$$

which can be integrated to obtain

$$\Phi = T\sqrt{-\gamma}. \tag{17}$$

The integration constant $T$ has indeed the meaning of string tension.

All the above can be straightforwardly generalized to branes. Indeed, the relevant action for a $p$-brane is

$$S = S_p + S_{p-\mathrm{gauge}}, \tag{18}$$

where

$$S_p = -\int d^{p+1}\sigma \Phi \gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} \tag{19}$$

and

$$S_{p-\mathrm{gauge}} = \int d^{p+1}\sigma \Phi \frac{\epsilon^{a_1 \ldots a_{p+1}}}{\sqrt{-\gamma}} \partial_{[a_1} A_{a_2 \ldots a_{p+1}]} \tag{20}$$

with $\Phi$ now defined in terms of $p+1$ scalar fields as

$$\Phi = \epsilon^{a_1 \ldots a_{p+1}} \epsilon_{j_1 \ldots j_{p+1}} \partial_{a_1}\varphi^{j_1} \ldots \partial_{a_{p+1}}\varphi^{j_{p+1}}; \tag{21}$$

this *kind* of $p$-brane, obtained with the modified measure $\Phi$, is only globally scale invariant. In order to obtain conformal invariance for $p > 1$ one should use a *vector* gauge field $A_a$, instead of the antisymmetric tensor field $A_{a_1 \ldots a_p}$ (see [5]). In this paper we will study, however, the globally scale invariant case (18), (19), (20). There, the variation with respect to $A_{a_1 \ldots a_p}$ gives

$$\epsilon^{a_1 \ldots a_{p+1}}\partial_{a_1} \left(\frac{\Phi}{\sqrt{-\gamma}}\right) = 0, \tag{22}$$

which again means

$$\Phi = T\sqrt{-\gamma}, \tag{23}$$

where $T = \text{const.}$ is the *dynamically generated* brane tension.

The equation of motion obtained from the variation of the $\varphi^j$ fields gives (if $\Phi \neq 0$)

$$-\gamma^{cd}\partial_c X^\mu \partial_d X^\nu g_{\mu\nu} + \frac{\epsilon^{a_1\ldots a_{p+1}}}{\sqrt{-\gamma}}\partial_{[a_1}A_{a_2\ldots a_{p+1}]} = M; \tag{24}$$

as in the string case, solving for the last term on the left-hand side and considering also the equation obtained from the variation with respect to $\gamma_{ab}$, one obtains that

$$\gamma_{ab} = \frac{1-p}{M}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu}. \tag{25}$$

If $M < 0$ (since $p > 1$), then by rescaling of the metric $\gamma_{ab}$ one can obtain simply that $\gamma_{ab}$ is the induced metric on the brane.

We see that the action (18) reproduces the normal brane dynamics: the crucial difference is that now the tension becomes a dynamical one.

## III. COUPLING OF STRINGS AND BRANES TO EXTERNAL SOURCES

### A. Strings and Branes "cutting"

If to the action of the brane (18) we add a coupling to a world-sheet current $j^{a_2\ldots a_{p+1}}$, i.e. a term

$$S_{\text{current}} = \int d^{p+1}\sigma A_{a_2\ldots a_{p+1}} j^{a_2\ldots a_{p+1}}, \tag{26}$$

then the variation of the total action with respect to $A_{a_2\ldots a_{p+1}}$ gives

$$\epsilon^{a_1\ldots a_{p+1}}\partial_{a_1}\left(\frac{\Phi}{\sqrt{-\gamma}}\right) = j^{a_2\ldots a_{p+1}}. \tag{27}$$

We thus see indeed that, in this case, the dynamical character of the tension becomes very much relevant.

If, for example, we consider distributional sources of the *delta* function type, we can get finite discontinuities in the tension. For simplicity let us consider the string example, where a source of the form

$$j^0(\sigma) = T\delta(\sigma^1), \quad j^1(\sigma) = 0 \tag{28}$$

is considered. Then the equation

$$\epsilon^{ab}\partial_b\left(\frac{\Phi}{\sqrt{-\gamma}}\right) = j^a \tag{29}$$

has a solution

$$\frac{\Phi}{\sqrt{-\gamma}} = T\theta(\sigma^1), \tag{30}$$

i.e., zero string tension for $\sigma^1 < 0$ and finite string tension for $\sigma^1 > 0$. That is, by means of the delta function source we have *cut* the string. Point like sources of this type were considered in [3]. Generalizations of this to higher dimensional branes are straightforward.

Again in the string case, we will see that a new feature is obtained by considering a current of the form

$$j^0(\sigma) = 0, \quad j^1(\sigma) = -T\delta(\sigma^0); \tag{31}$$

indeed, now equation (29) gives

$$\frac{\Phi}{\sqrt{-\gamma}} = T\theta(\sigma^0), \tag{32}$$

which represents the *creation* of the string tension at the instant $\sigma^0 = 0$ from the action of the external source. Notice that both the sources, (28) and (31), obey the continuity equation.

As we will see in the following subsection III B, a source of the form (31) can be of importance to model part of the process of pair creation in the world-sheet.

### B. Pair creation

A piecewise combination of sources of the type (28) and (31) can be used to represent pair creation. Take, as in figure 1(a), a source of the form (please, note that, for simplicity, in this subsection we are going to use $\tau$ in place of $\sigma^0$, $\sigma$ in place of $\sigma^1$, so that $\sigma^a \equiv (\tau, \sigma)$, and that this very remark also applies to figure 1):

$$\begin{aligned} j^0 &= T\theta(\tau)\left(\delta(\sigma-\sigma_1) - \delta(\sigma-\sigma_2)\right) \\ j^1 &= -T\delta(\tau)\theta(\sigma-\sigma_1)\theta(\sigma_2-\sigma), \end{aligned}$$

with $\sigma_2 > \sigma_1$. In this case the solution of (29) gives (see figure 1(b))

$$\frac{\Phi}{\sqrt{-\gamma}} = T_0 + T$$

for the region $\tau > 0$, $\sigma_1 < \sigma < \sigma_2$, and

$$\frac{\Phi}{\sqrt{-\gamma}} = T_0$$

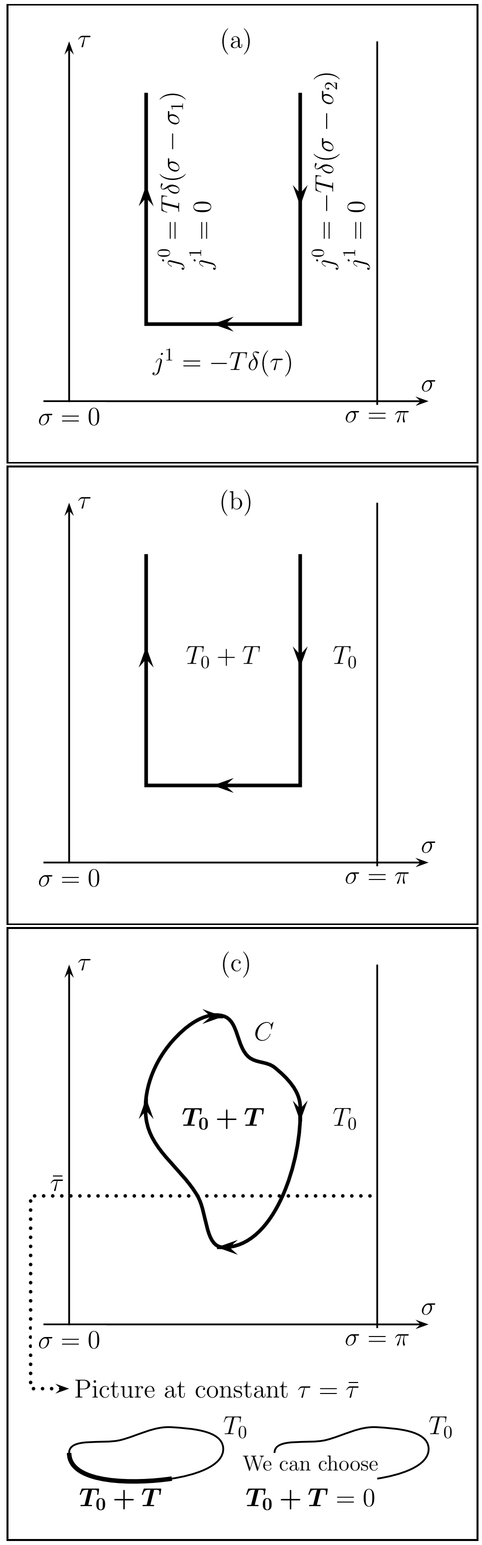


FIG. 1: Some examples of models with dynamical tension, as discussed in the main text, are presented. For example in (c) it is shown how a string with a zero-tension segment can be obtained (please, see the main text for details).

elsewhere, $T_0$ being an integration constant.

In the case of a pair creation and annihilation, treated in general, the current can be described as (figure 1(c))

$$j^a = T \oint_C d\sigma^a \delta^{(2)}(\sigma^i - \sigma^i(\lambda)),$$

where $\sigma^i = \sigma^i(\lambda)$ is a parametrization of $C$. Then the solution of (29) is

$$\frac{\Phi}{\sqrt{-\gamma}} = T_0 + T$$

for points inside $C$ and

$$\frac{\Phi}{\sqrt{-\gamma}} = T_0$$

for points outside $C$, again $T_0$ being an integration constant. By properly choosing the integration constant, as shown in figure 1(c), a string with a zero tension segment can be obtained. In more general cases, we can also obtain strings with segments of different tension.

### C. Coupling to an external (scalar) field

#### 1. *General discussion*

Suppose that we have an external scalar field $\phi(x^\mu)$ defined in the bulk. From this field we can define the induced conserved world-sheet current

$$j^{a_1 \ldots a_{p+1}} = g\partial_\mu \phi \frac{\partial X^\mu}{\partial \sigma^a} \epsilon^{a a_2 \ldots a_{p+1}} \equiv g \partial_a \phi \epsilon^{a a_2 \ldots a_{p+1}}, \tag{33}$$

where $g$ is some coupling constant.

Then (27) can be integrated to obtain

$$\frac{\Phi}{\sqrt{-\gamma}} = g\phi + T_0, \tag{34}$$

which describes the transfer of energy from the bulk scalar field $\phi$ to the brane. Notice that it is not hard to obtain from the string point of view a current of the form (31), which would create the tension of the string from zero tension: a scalar field which is a step function of the time could do this.

*2. Symmetries of the system*

The part of the action which contains the gauge fields $A_{a_2 \ldots a_{p+1}}$, after integration by parts, can be expressed as

$$\int d^{p+1}\sigma \epsilon^{a_1 \ldots a_{p+1}} \left( \frac{\Phi}{\sqrt{-\gamma}} - g\phi \right) \partial_{a_1} A_{a_2 \ldots a_{p+1}}, \tag{35}$$

where we remark for clarity the notation that we are using: *world-sheet* scalar fields $\varphi^j$ appear in the measure $\Phi$, *whereas* the unindexed $\phi$ denotes a *bulk* scalar field. This action is of course invariant under the conventional gauge transformations

$$A_{a_2 \ldots a_{p+1}} \quad \longrightarrow \quad A_{a_2 \ldots a_{p+1}} + \partial_{[a_2} \Lambda_{a_3 \ldots a_{p+1}]}. \tag{36}$$

In addition, there also is the highly non conventional infinite dimensional symmetry

$$A_{a_2 \ldots a_{p+1}} \quad \longrightarrow \quad A_{a_2 \ldots a_{p+1}} + \epsilon_{a a_2 \ldots a_{p+1}} f^a \left( \frac{\Phi}{\sqrt{-\gamma}} - g\phi \right), \tag{37}$$

where $\epsilon_{a a_2 \ldots a_{p+1}}$ is numerically the same as $\epsilon^{a a_2 \cdots a_{p+1}}$, i.e. the alternating symbol in $p$ dimensions. In this case, using the identity

$$\epsilon^{a_1 a_2 \ldots a_{p+1}} \epsilon_{a a_2 \ldots a_{p+1}} = \frac{1}{p!} \delta^{a_1}_{a},$$

we obtain that the integrand in (35) is transformed by

$$\frac{1}{p!} f^a \left( \frac{\Phi}{\sqrt{-\gamma}} - g\phi \right) \partial_a \left( \frac{\Phi}{\sqrt{-\gamma}} - g\phi \right), \tag{38}$$

which equals $\partial_a I^a$, $I^a$ being the integral of the function $f^a$ with respect to its argument. (38) is therefore a total derivative and (37) is a symmetry.

This kind of symmetry would be absent if we would have introduced a *field strength square term*, i.e. a kinetic term for the world-sheet gauge field, and therefore it provides a good argument for keeping linearity in $\partial_{[a_1} A_{a_2 \ldots a_{p+1}]}$ in the form of the action.

One should notice that a symmetry of this kind will exist in all situations in which one considers an antisymmetric tensor field $A_{a_2 \ldots a_{p+1}}$ coupled to a Lagrangian $\mathcal{L}_1$, which depends on dynamical variables different from $A_{a_2 \ldots a_{p+1}}$, in the form

$$S_1 = \int d^{p+1}\sigma \epsilon^{a_1 \ldots a_{p+1}} \partial_{a_1} A_{a_2 \ldots a_{p+1}} \mathcal{L}_1. \tag{39}$$

Then there is the symmetry

$$A_{a_2 \ldots a_{p+1}} \quad \longrightarrow \quad A_{a_2 \ldots a_{p+1}} + \epsilon_{a a_2 \ldots a_{p+1}} f^a(\mathcal{L}_1). \tag{40}$$

This is true whether or not (39) refers to a part of the action in a $p$-brane or in a gravitational theory (i.e. it represents a bulk gravitational action). In this case a similar type of symmetry was discussed in [4], where instead of the density $\epsilon^{a_1 a_2 \dots a_{p+1}} \partial_{a_1} A_{a_2 \dots a_{p+1}}$ the form (2) was considered. Then, given a coupling $\Phi \mathcal{L}_1$ it was shown that

$$\varphi^i \quad \longrightarrow \quad \varphi^i + f^i(\mathcal{L}_1)$$

was a symmetry.

In the case of the action (18), (19), (20), and even when adding the coupling (26) where the current does not involve the $\varphi^i$ fields, there is also the symmetry

$$\phi^j \quad \longrightarrow \quad \phi^j + f^j(\mathcal{L}_1) \tag{41}$$

where

$$\mathcal{L}_1 = -\gamma^{ab} \partial_a X^\mu \partial_b X^\nu + \frac{\epsilon^{a_1 \dots a_{p+1}}}{\sqrt{-\gamma}} \partial_{a_1} A_{a_2 \dots a_{p+1}}.$$

This symmetry holds as long as there are no $\Phi^2$, $\Phi^3$, $\dots$ terms in the action, i.e. $\Phi$ must appear only linearly in the action.

A straightforward calculation of the Noether currents derived from the transformation (40) allows us to see that they are

$$j^a = \frac{1}{p!} \mathcal{L}_1 f^a(\mathcal{L}_1);$$

indeed, since as a consequence of the equations of motion $\mathcal{L}_1 = \text{const.}$ we see that all components of $j^a$ are constant and $\partial_a j^a = 0$ is true on mass-shell.

### *3. Discussion of possible cosmological applications*

Here we have discussed a model by mean of which it is possible to transfer energy from a scalar field to a system of strings and/or branes and *viceversa*.

Therefore very tiny strings or branes (with negligible tension) could eventually fatten to become extended objects with greater tension. This classical process, by means of which the energy is transferred from the scalar field to the extended object, can replace to some extent the reheating process in the inflationary universe: indeed, here we only need a very small amount of energy density of strings or branes, which could then get amplified by the process.

In the context of the exchange of energy density of vacuum and dark matter (of which the extended objects could be taken as models), we could obtain models whereby the *dark matter* and *dark energy* interact.

## IV. BUBBLE CREATION AND MASS GENERATION

Consider now a 2-brane, embedded in 4-dimensional spacetime, which has a dynamical tension governed by $\Phi/\sqrt{-\gamma}$. This brane has the possibility to couple to a bulk three index antisymmetric field strength $A_{\lambda\mu\nu}$. For example this coupling could be of the form

$$e\int d^3\sigma A_{\lambda\mu\nu}(X(\sigma))\partial_a X^\lambda \partial_b X^\mu \partial_c X^\nu \epsilon^{abc}, \tag{42}$$

which can be written also as

$$e\int d^4x \int d^3\sigma A_{\lambda\mu\nu}(x)\partial_a X^\lambda \partial_b X^\mu \partial_c X^\nu \epsilon^{abc}\delta^{4)}(x^\alpha - X^\alpha(\sigma))$$

$e$ being some coupling constant.

This type of coupling is invariant under the gauge transformation of the bulk field $A_{\lambda\mu\nu}$

$$A_{\lambda\mu\nu} \quad\longrightarrow\quad A_{\lambda\mu\nu} + \partial_{[\lambda}\Lambda_{\mu\nu]}. \tag{43}$$

In this case, the dynamical tension (i.e. the factor $\Phi$ present in the other parts of the action) does not appear in the coupling of the antisymmetric gauge field $A_{\lambda\mu\nu}$ to the world-sheet current

$$J^{\alpha\beta\gamma} = e\int d^3\sigma \epsilon^{abc}\partial_a X^\alpha \partial_b X^\beta \partial_c X^\gamma \delta^{4)}(x^\mu - X^\mu(\sigma)).$$

If we now decide that the coupling to the external gauge field must be part of the action which is weighted by the measure $\Phi$, we find at first a problem with the gauge invariance (43), since the coupling

$$\begin{aligned} &e\int d^4x \int d^3\sigma \frac{\Phi}{\sqrt{-\gamma}} \times \\ &\times A_{\lambda\mu\nu}\partial_a X^\lambda \partial_b X^\mu \partial_c X^\nu \epsilon^{abc}\delta^{4)}(x^\mu - X^\mu(\sigma)) \end{aligned} \tag{44}$$

is not invariant anymore under (43).

Another way to see the failure of the gauge invariance in the case (44), is by noticing that the current

$$J^{\lambda\mu\nu}_\Phi = e\int d^3\sigma \frac{\Phi}{\sqrt{-\gamma}} \epsilon^{abc}\partial_a X^\lambda \partial_b X^\mu \partial_c X^\nu \delta^{4)}(x^\alpha - X^\alpha(\sigma)) \tag{45}$$

fails to be conserved, since it is multiplied by the dynamical tension $\Phi/\sqrt{-\gamma}$.

If $\Phi/\sqrt{-\gamma}$ starts from zero, this represents indeed the creation of the 2-brane current from *nothing*. The way to restore gauge invariance, even though we couple to a non conserved current, is to introduce the compensating auxiliary Stückelberg field and consider, instead of (44), the coupling

$$e\int d^4x\int d^3\sigma\frac{\Phi}{\sqrt{-\gamma}}\delta^{4)}(x^\alpha - X^\alpha(\sigma))\times \tag{46}$$

$$\times\left(A_{\lambda\mu\nu}+\partial_{[\lambda}\Lambda_{\mu\nu]}\right)\partial_a X^\lambda\partial_b X^\mu\partial_c X^\nu\epsilon^{abc}. \tag{47}$$

This is analogous to what is done in [6], although there only the situation in which the brane is abruptly created at some spacelike surface was considered (as, for instance, in some Minkowski description of a nucleation process).

In this case, the factor $\Phi/\sqrt{-\gamma}$ could be continuous (i.e. it is not bound to be just some theta function) and represent a continuous build up, or growth, of the 2-brane world sheet current, together with the brane tension.

The theory of the three index field strength coupled with a non conserved current is not consistent if only an ordinary kinetic term proportional to

$$\int d^4x\sqrt{-g}\left(\partial_{[\lambda}A_{\mu\nu\rho]}\right)^2$$

is considered. Consistency is restored, however, if a mass term

$$m^2\int d^4x\sqrt{-g}\left(A_{\lambda\mu\nu}+\partial_{[\lambda}A_{\mu\nu]}\right)^2$$

is added to the action in addition to the previous kinetic term. Then

$$m^2\partial_\lambda\left(A^{\lambda\mu\nu}+\partial^{[\lambda}\Lambda^{\mu\nu]}\right)=\partial_\lambda J_\Phi^{\lambda\mu\nu} \tag{48}$$

follows. This is a basic relation between the divergence of $A_{\lambda\mu\nu}+\partial_{[\lambda}A_{\mu\nu]}$ and the divergence of the current $J_\Phi^{\lambda\mu\nu}$. If $m^2\to 0$ equation (48) becomes a contradiction since it forces $J_\Phi^{\lambda\mu\nu}$ to be conserved, while we know that if $\Phi/\sqrt{-\gamma}$ has a non-trivial spacetime dependence this is not the case. However $m\neq 0$, i.e. mass generation restores consistency.

## V. DISCUSSION, CONCLUSIONS AND OUTLOOK

Here we have seen what the role of dynamical tension may be in the process of strings and branes creation. The dynamics of the string/brane tension is governed by world-sheet

currents. In particular these world-sheet currents could be generated by lower dimensional objects living in the extended objects, for example point particles in a string.

In the case of a string we have studied the discontinuities in the string tension produced by these point particles and the effect induced on the string tension by the process of pair creation taking place in the string world-sheet.

We have also studied effects by means of which energy may be transferred from the bulk to strings and branes. This is possible because, for example, a bulk scalar field induces naturally a world-sheet current that affects the tension.

A process of this kind could be of interest in the braneworld scenario., where energy that is transferred to the bulk could appear as "missing" from our 4-dimensional world.

Other applications could be in the field of cosmology, where the classical problem of reheating in the inflationary scenario involves the transfer of energy from a scalar field to matter. In our model, this is realized by the transfer of energy from a bulk scalar field to extended objects, by altering their tension. We could thus have a situation in which a string or brane with, initially, a very tiny tension, could grow very much by means of the above mechanism.

It is possible to consider the action of a 2-brane weighted by he measure $\Phi$, including the coupling to external bulk fields $A_{\lambda\mu\nu}$. If this is done in the way that have we described in this paper, it is also possible to consider the creation or "growth" of the current coupled to $A_{\lambda\mu\nu}$. In this case the current is *not* conserved. Restoration of the gauge invariance $A_{\lambda\mu\nu} \rightarrow A_{\lambda\mu\nu} + \partial_{[\lambda}\Lambda_{\mu\nu]}$ by considering Stückelberg compensating fields is possible and gives rise to a consistent model if we include mass generation for the $A_{\lambda\mu\nu}$ fields.

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[1] For reviews on string and branes as theories of fundamental interactions see M. Green, J. Schwarz and E. Witten, *Superstring Theory*, Vols. 1, 2, Cambridge University Press (1987); J. Polchinski, *String Theory*, Vols. 1, 2, Cambridge University Press (1988); C. Johnson, *D-branes*, Cambridge University Press (2002) and references therein (see also hep-th/0007170); Y. Ne'eman and E. Eizenberg, *Membranes and Other Extendons*, World Scientific (1995).

[2] A. Vilenkin, Phys. Rept. **121**, 263 (1985)

[3] E. I. Guendelman, Class. Quantum Grav. **17**, 3673 (2000) (hep-th/0005041); E. I. Guendelman, Phys. Rev. D **63**, 046006 (2001) (hep-th/0006079); E. I. Guendelman, A. B. Kaganovich, E. Nissimov and S. Pacheva, Phys. Rev. D **66**, 046003 (2002) (hep-th/0203024); E. I. Guendelman, A. B. Kaganovich, E. Nissimov and S. Pacheva, in "First Workshop on Gravity, Astrophysics and Strings", P. Fiziev *et al.* eds., Sofia University Press (2003) (hep-th/0210062); E. I. Guendelman, A. B. Kaganovich, E. Nissimov and S. Pacheva, in "Second International School on Modern Mathematical Physics", Kopaonik (Serbia and Montenegro), B. Dragovich and B. Sazdovich eds., Belgrade Institute of Physics Press (2003) (hep-th/0304269); E. I. Guendelman, A. B. Kaganovich, E. Nissimov and S. Pacheva, in "Lie Theory and its Applications in Physics", V. Dobrev and H. Doebner eds., World Scientific (2004) (hep-th/0401083);

[4] E. I. Guendelman and A. B. Kaganovich, Phys. Rev D **53**, 7020 (1996); E. I. Guendelman and A. B. Kaganovich, Phys. Rev D **60**, 065004 (1999) (hep-th/9905029); E. I. Guendelman, Mod. Phys. Lett. A **14**, 1043 (1999) (hep-th/9901017); E. I. Guendelman, Class. Quantum Grav. **17**, 361 (2000); E. I. Guendelman and A. B. Kaganovich, Int. J. Mod. Phys. A **17**, 417 (2002) (hep-th/0106152); E. I. Guendelman and A. B. Kaganovich, Mod. Phys. Lett. A **17**, 1227 (2002) (hep-th/0110221, gr-qc/0312006, hep-th/0411188); E. I. Guendelman, Phys. Lett. **580B**, 87 (2004) (gr-qc/0303048); E. I. Guendelman and E. Spallucci, Phys. Rev. D **70**, 026003 (2004) (hep-th/0311102); E. I. Guendelman, hep-th/0507092.

[5] E. I. Guendelman, A. B. Kaganovich, E. Nissimov and S. Pacheva, to appear in the "Second Workshop on Gravity, Astrophysics and Strings", P. Fiziev *et al.* eds., Sofia University Press (2005) (hep-th/0409078); E. I. Guendelman, A. B. Kaganovich, E. Nissimov and S. Pacheva, hep-th/0409208; E. I. Guendelman, A. B. Kaganovich, E. Nissimov and S. Pacheva, to appear

in Phys. Rev. D hep-th/0507193; E. I. Guendelman, A. B. Kaganovich, E. Nissimov and S. Pacheva, to appear in the proceedings of the "Third International School on Modern Mathematical Physics", Zlatibor (Serbia and Montenegro), B. Dragovich and B. Sazdovich eds., Belgrade Institute of Physics Press (2005) (hep-th/0501220);

[6] S. Ansoldi, A. Aurilia and E. Spallucci, Phys. Rev. D **64**, 025008 (2001).